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2005 J. Phys. A: Math. Gen. 38 3057

(http://iopscience.iop.org/0305-4470/38/13/016)

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND GENERAL

J. Phys. A: Math. Gen. **38** (2005) 3057–3063 doi:10.1088/0305-4470/38/13/016

The structure of a two-dimensional magnetic dusty plasma*

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Received 24 September 2004, in final form 11 January 2005
Published 14 March 2005
Online at stacks.iop.org/JPhysA/38/3057

Abstract

The structure of a two-dimensional dusty plasma in an external magnetic field has been investigated in detail by molecular dynamics simulation. The pair correlation function, mean square displacement, the static structure factor and the bond angle correlation function have been calculated to characterize the structural properties of the dusty plasma for different values of coupling constant and magnetic field strength. The results show that the dusty plasma system with a fixed coupling constant can have a phase transition from gas to fluid, and from fluid to a solid state, when the external magnetic field strength is increased to a critical value. The critical magnetic field strength generally decreases with increasing coupling constant in the system.

PACS numbers: 52.27.−h, 52.27.Lw, 52.65.Yy

1. Introduction

The investigation of dusty plasmas has attracted considerable attention. A dusty plasma is an ionized gas containing dust particles which may acquire electric charges in the plasma environment and interact with each other. The dynamical behaviour of the charged particles can be affected greatly by the external electric field and/or magnetic field. In the last few years, many theoretical, experimental and simulational research works have been devoted to studying the structural property of an unmagnetized dusty plasma [1–7]. However, space and laboratory dusty plasmas are usually confined in an external magnetic field [8], but the theoretical and simulational studies of a magnetized dusty plasma are quite open so far. In this paper, we intend to investigate the structural properties of a two-dimensional (2D) dusty plasma system in different external magnetic fields using molecular dynamics (MD) simulation.

* Supported by the National Natural Science Foundation of China under grant nos. 10175036 and 10205025.

2. Physical model and molecular dynamics simulation

In the simulations, we consider a 2D dusty plasma system which is composed of many identical dusty particles with charge Q and mass M immersed in a neutral background. In the 2D system, the particles interact with each other through a Yukawa or a screened Coulomb potential $\phi(r) = (Q/4\pi\epsilon_0 r)\exp(-r/\lambda_D)$, where r is the interparticle spacing, and λ_D is the screening length. In order to investigate the effects of an external magnetic field on the behaviour of the particles, we suppose a magnetic field is introduced along the direction perpendicular to the 2D plane of the dusty plasma system. The force $\vec{F}_i(t)$ acted on the ith particle is given by

$$\vec{F}_i(t) = -Q\sum_{j\neq i}\nabla_i\phi(r_{ij}) + Q\vec{v}_i(t)\times\vec{B}, \tag{1}$$

where $\phi(r_{ij})$ is the interaction potential between particles i and j which are situated at $\vec{r}_i$ and $\vec{r}_j$, respectively. $r_{ij} = |\vec{r}_i - \vec{r}_j|$ is the distance between particles i and j and $\vec{v}_i(t)$ is the velocity of particle i at time t. Normally, the thermodynamics of the system can be characterized by two dimensionless parameters: $\Gamma = Q^2/(4\pi\epsilon_0 ak_BT)$ and $\kappa = a/\lambda_D$, where a is the mean interparticle distance and T is the system temperature. In this paper, the external magnetic field B is normalized by $B_0 = \sqrt{M/4\pi\epsilon_0 a^3}$.

In this research, we use constant temperature MD to simulate the structural properties of a 2D dusty plasma. The constant temperature is achieved by the Nosé–Hoover thermostat scheme [9]. The simulations are performed in a system with 256 particles in a 2D square box of side $L = 16a$ under periodic boundary conditions. The interaction of a given particle i with other particles j and the periodic images of particles j in other neighbour boxes is considered to calculate the electrostatic potential in the MD simulations. A cut-off distance L of the pair interaction potential is also chosen to save computing time for the short-range screening Coulomb potential. The time step is $0.1\omega_{\mathrm{pd}}^{-1}$ (here ω_{pd} is the dusty plasma frequency). In each simulation, the initial run lasts 5×10^4 steps for achieving system equilibrium, and in subsequent 3×10^4 time steps, the structural properties of the system are measured by calculating the mean square displacement (MSD) $\langle r^2(t)\rangle$, pair correlation function $g(r)$, static structure factor $S(q)$ and bond angle correlation function $G_6(r)$. These functions can be given as follows [6, 10]:

$$g(r) = \frac{S}{N}\frac{N(r,\Delta)}{2\pi r\Delta}, \tag{2}$$

$$\langle r^2(t)\rangle = \left\langle \frac{1}{N}\sum_{i=1}^{N}(\vec{r}_i(t) - \vec{r}_i(0))^2 \right\rangle, \tag{3}$$

$$S(q) = \frac{1}{N}\left\langle \sum_{i,j}\exp[\mathrm{i}\vec{q}\cdot\vec{r}_{ij}] \right\rangle, \tag{4}$$

$$G_6(r) = \langle \exp\{\mathrm{i}6[\theta(\vec{r}) - \theta(0)]\}\rangle. \tag{5}$$

Here N is the number of simulation particles, S is the area of the simulated region, $N(r,\Delta)$ is the number of particles located between $r - \Delta/2$ and $r + \Delta/2(\Delta = 0.1a)$, $\langle\cdots\rangle$ denotes the thermal average in equation (3), and denotes thermal average and angle average over the direction of wave vector $\vec{q}$ in equation (4). $q = |\vec{q}| = |\vec{k}a|$ is the reduced wave number, $\vec{r}_i(t)$ and $\vec{v}_i(t)$ are the position and velocity of the ith particle at time t, $\vec{r}_{ij} = \vec{r}_i - \vec{r}_j$ is the displacement between particles i and j, $\theta(r)$ is the angle made by a bond between a particle located at $\vec{r}$ with its nearest neighbours with an arbitrary fixed axis. In this paper, the

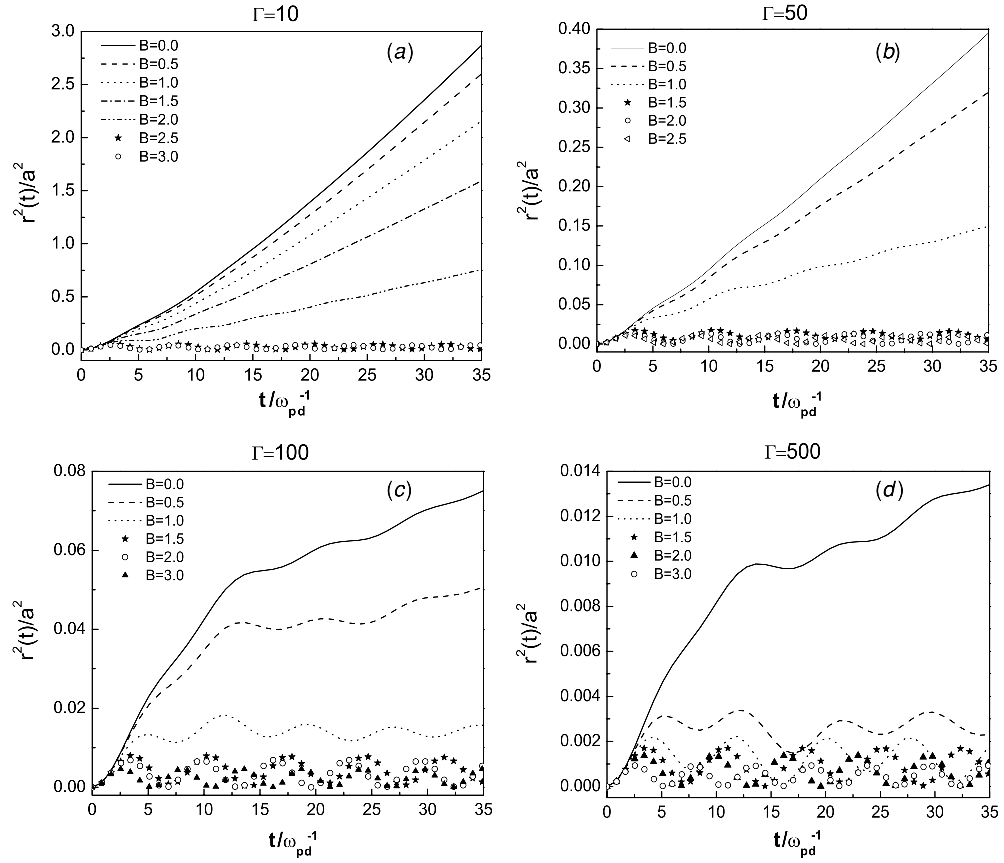

Figure 1. Mean square displacement variations with time for different magnetic strengths and four values of Γ.

length, time, energy and magnetic field strength are normalized by a, $t_0 = \sqrt{4\pi\epsilon_0 M a^3/Q^2} = \sqrt{4\pi}\omega_{\mathrm{pd}}^{-1}$, $U_0 = Q^2/4\pi\epsilon_0 a$ and $B_0 = \sqrt{M/4\pi\epsilon_0 a^3}$, respectively. In the simulations, the screening strength κ is chosen as $\kappa = 1$ to present real experimental conditions.

3. Numerical results and discussions

The simulations are performed with the coupling parameter Γ ranging from 10 to 500 and with the normalized magnetic field strength B ranging from 0.0 to 5.0. In [6], the researchers studied the effects of coupling constant Γ on the structures of a 2D dusty plasma without magnetic field, and they found that the 2D system has an obvious change in the structure, even having phase transitions from gas to fluid and from fluid to solid state when Γ is increased continuously. Now we intend to study the effects of an external magnetic field on the structure of a 2D dusty plasma system with different Γ.

Figure 1 presents the particle's mean square displacement at different values of coupling constant Γ and magnetic field strength B. From figures 1(a)–(d), one can see that at large time (e.g., when $t = 30\omega_{\mathrm{pd}}^{-1}$), the MSD is decreased greatly with increasing B at a given Γ, and when B is increased higher than a certain value, the MSD does not decrease further, and exhibits an oscillatory feature with time. For example, in figure 1(b), the MSD $\langle r^2(t)\rangle \propto t^{1.068}$

at $B = 0.5$, and $\langle r^2(t) \rangle \propto t^{0.870}$ at $B = 1.0$; when $B \geqslant 1.5$, the MSD does not decrease obviously with B, and exhibits an oscillatory property with time. The explanation for these phenomena is as follows: in a system with magnetic field the charged particles will gyrate, and when the magnetic field strength is high enough, the gyro-radius becomes very small, so the particles are restricted in a small region around their equilibrium positions and cannot move freely a long distance, and the MSD naturally exhibits an oscillation property. For example, when $\Gamma = 100$ and $B = 3.0$, the gyro-radius $\rho = 0.0471a$ (here ρ is calculated from $\rho = Mv/(QB)$, where v is particle's average speed estimated from the system temperature), and the system is in a solid-like state. This oscillation property is similar to the MSD results in a 2D dusty plasma system without magnetic field but at a large Γ [6, 7]. In [7] when Γ is high enough, for example, $\Gamma = 10\,000$, the system becomes a perfect crystal, so the particles just vibrate around their equilibrium positions, and the MSD shows oscillation properties.

From figures 1(*a*) and (*b*), one can also see that the MSD increases gradually with time when the magnetic field B is lower than a certain value ($B = 2.5$ and 1.5) at a given Γ (e.g. $\Gamma = 10$ and 50) which indicates the system is in a gas or fluid state. So, based on the above discussions, a phase transition must occur at $B = 2.5$ and 1.5 in the 2D dusty plasma system with $\Gamma = 10$ and 50, respectively. From figures 1(*c*) and (*d*), one can see that when the magnetic field is lower than a certain value (e.g. $B = 1.0$ and 0.5) at a given Γ (e.g. $\Gamma = 100$ and 500), the slope of MSD changes obviously at $t = 13\omega_{\mathrm{pd}}^{-1}$. According to subdiffusive regime [11], we can see the system is in a fluid state. With increasing magnetic field B, the system will also make a phase transition from fluid to solid state. From above discussions, we can conclude that the external magnetic field has strong effects on the particles' dynamical behaviour, and the system has a phase transition from fluid to solid state with increasing magnetic field strength. The critical magnetic field strength decreases with increasing coupling constant Γ.

The structure of a dusty plasma system can also be analysed by the pair correlation function $g(r)$, which is defined as the probability of finding a pair of particles at a distance r apart. Figure 2 shows the $g(r)$ variations for different magnetic field strengths B and coupling constants Γ. The numbers marked with the arrows indicate the number of particle in successive shells. From figure 2, one can see that with increasing magnetic field $g(r)$ has obvious changes at a given Γ. When $\Gamma = 10$ and at $B = 2.0$, several peaks appear in $g(r)$ indicating that the particles are distributed in specific shells around any test particle. In this case, each particle has six nearest neighbours, twelve next neighbours and eighteen next–next neighbours. In other words, the neighbour number is in a shell structure (6-12-18), which indicates that the system is in a liquid state. When $B \geqslant 2.5$, there appears a shoulder on each peak except the first one, and the third peak becomes higher than the second one, which indicates the system starts to freeze. At $B = 5.0$, the $g(r)$ shows a crystalline structure, and each particle has neighbours of shell structure (6-6-6-12-6), in successive shells. So, one can see that the external magnetic field causes a obvious phase transition from fluid to solid state at $\Gamma = 10$. For the cases of $\Gamma = 25$, 50 and 62.5, the structure of $g(r)$ also changes obviously with increasing magnetic field, for example, when $\Gamma = 25$ and 50 the $g(r)$ has an obvious change at $B = 2.0$ and 1.5, respectively. Based on above discussions, we can see that the magnetic field causes a phase transition from fluid to solid state at a critical magnetic field strength in the system. According to [6], increasing coupling constants Γ can also cause a phase transition in a 2D dusty plasma system from fluid to solid state. Figures 3(*a*) and (*b*) are the enlargements of the $g(r)$ shown in figure 2 with different magnetic field strengths B at $\Gamma = 10$ and with different coupling constants at $B = 2.0$, respectively, in order to clearly compare the effects of magnetic field and coupling constants on the system structure property. From figrues 3(*a*) and (*b*), we can also clearly compare the effect of parameters B and Γ on the system structure: for the case of fixing

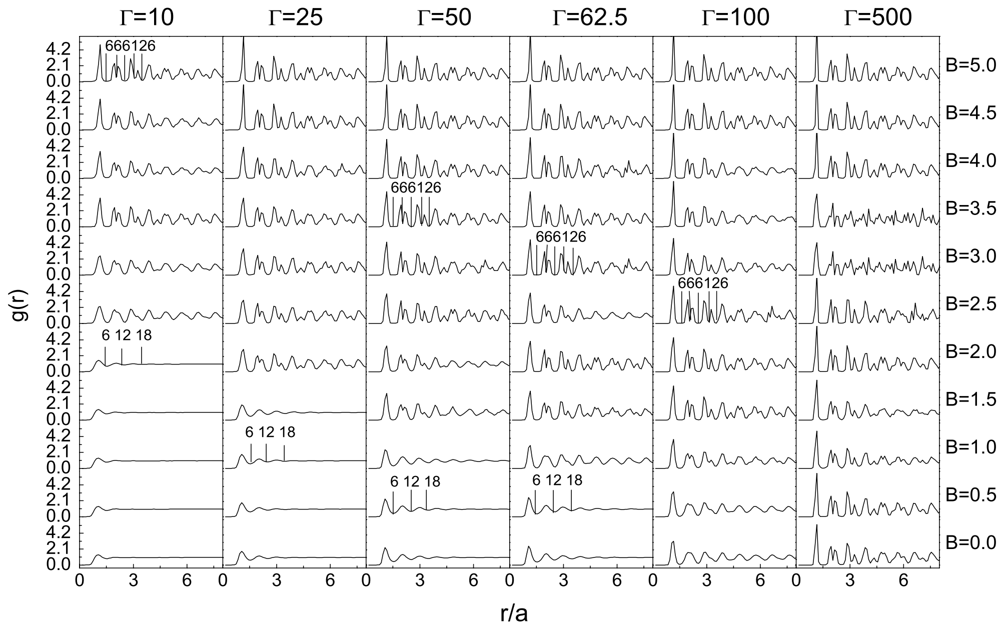

Figure 2. Pair correlation function $g(r)$ versus r/a for different values of B and Γ.

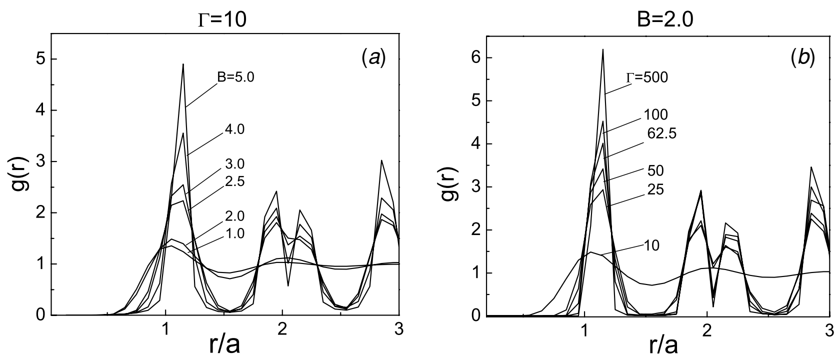

Figure 3. Pair correlation function $g(r)$ versus r/a for different values of B at $\Gamma = 10$ (a) and for different Γ at $B = 2.0$ (b).

either B or Γ, the height (width) of the first peak in $g(r)$ increases (decreases) monotonously with another increasing parameter. So, we can see the increasing magnetic field B at a given Γ or increasing coupling constant Γ at a given B has very similar effects on the structural property of the 2D system.

The static structure factor $S(q)$ is another good indicator to distinguish a solid from a liquid. According to the Hansen–Verlet freezing criterion [12], the first peak of the static structure factor encodes the information of long-range ordering and consistently achieves a height of 2.85 when a three-dimensional (3D) system reaches freezing. The corresponding value in a 2D system is 4.0. The results of $S(q)$ at different magnetic field strengths B and $\Gamma = 10$ are shown in figure 4. From figure 4, one can see that the height of the first peak in $S(q)$ increases gradually with increasing magnetic field strength B, and when the magnetic field $B \geqslant 2.5$, the height of the peak is higher than 4.0 indicating the system already enters a

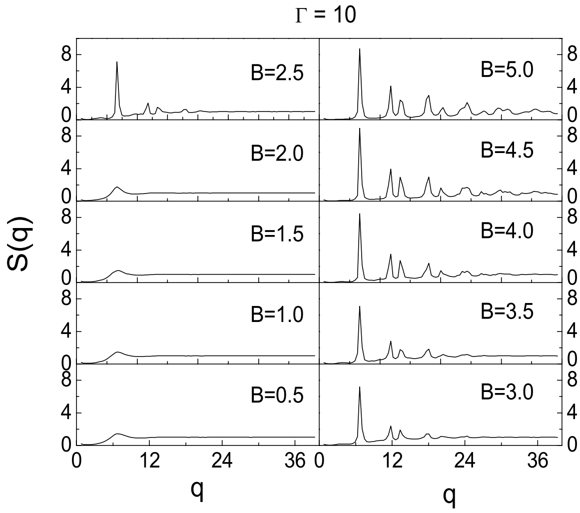

Figure 4. $S(q)$ variations with different values of B at $\Gamma = 10$.

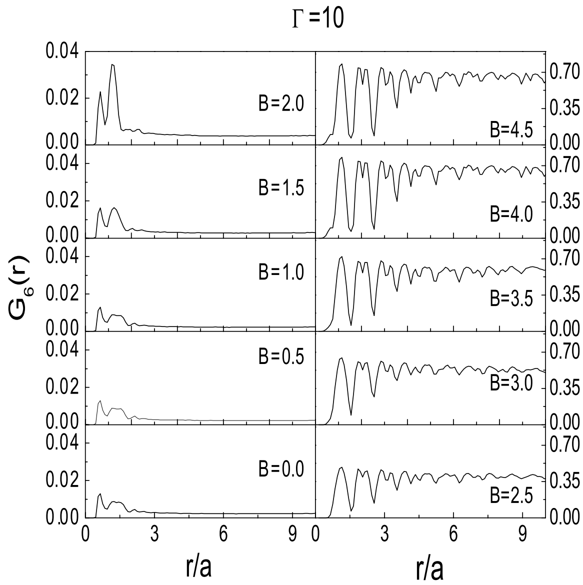

Figure 5. $G_6(r)$ variations with different values of B at $\Gamma = 10$.

solid state. In [6], the results show that with increasing coupling constant Γ, the system has a phase transition at a certain value of Γ. Our results show that increasing magnetic field can also have a similar effect on the system property.

A 2D solid shows a long-range bond-orientational order and a quasi-long-range positional order compared with a fluid, which is characterized by a short-range exponentially decaying order in both position and bond angle. The bond angle correlation function $G_6(r)$ measures the orientational order of the system structure. Figure 5 shows the $G_6(r)$ at $\Gamma = 10$ with different magnetic field strengths B. From figure 5, one can see that the bond-orientational order increases considerably with increasing B. At $B < 2.5$, the $G_6(r)$ decays exponentially to zero with r indicating the system is in the liquid state. At $B = 2.5$, the $G_6(r)$ decays much slower with r; At $B \geqslant 4.0$, the $G_6(r)$ tends to 0.8 with r, and a long-range bond-orientational order is presented, which indicates the system is in a solid state. In [6], the results of $G_6(r)$ also show that increasing Γ can also cause an obvious change in the structure of a 2D dusty plasma system without external magnetic field.

From above discussions of MSD, $g(r)$, $S(q)$ and $G_6(r)$, we can conclude that the applied external magnetic field can really cause an obvious change in the system structure, or even a phase transition from fluid to solid state can occur at a critical value of magnetic field.

4. Conclusions

In summary, we have studied the structure of a two-dimensional dusty plasma system under different external magnetic field strengths and coupling constants by molecular dynamics simulations. The simulations show that the applied magnetic field has an obvious effect on the system structure, and at a critical value of magnetic field strength a phase transition from a liquid to a solid state will occur in the system. The critical magnetic field strength decreases with increasing coupling constant in the system. The reason for the phase transition is that when the magnetic field is strong enough, the charged particles will gyrate with a small gyro-radius in the system, and the particles will be restricted (or localized) in a small region around their equilibrium positions. Thus, the 2D dusty plasma system is transformed from a fluid state into a solid state at a critical value of magnetic field strength.

Acknowledgment

The authors would like to thank Dr Zhaoyang Chen for supplying an effective method to calculate static structure factor and giving many helpful discussions.

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